VARIATIONAL EIGENSTRAIN MULTISCALE METHOD

Shaofan Li

Department of Civil and Environmental Engineering,
The University of California,
Berkeley, California, 94705
li@ce.berkeley.edu

A new subgrid (subscale) finite element method is proposed, which is termed as *the variational eigenstrain multiscale formulation*. It combines the essential ideas of Hughes' variational multiscale formulation with Eshelby's inclusion theory and Mura's equivalent eigenstrain principle.
The so-called variational multiscale method invented by Hughes (Hughes et al [1995][1998]) is a computational paradigm that is capable of dealing with multiscale phenomena. On the other hand, the so-called equivalent eigenstrain principle, which was established by Eshelby (Eshelby [1961]) and was later perfected by Mura (Mura [1987]), is a homogenization method in micromechanics that has been used in many engineering applications.
By synthesizing variational multiscale method with the equivalent eigenstrain principle, we have developed a new finite element formulation that can automatically homogenize its own discretization errors so that it may attain better accuracy in a coarse scale finite element computation than that of unhomogenized coarse scale computation. The paper provides the theoretical foundation of the method as well as a numerical example that validates the proposed variational eigenstrain multiscale method.
The basic idea of the new method is :
The residue strains of coarse scale solution are viewed as a form of eigenstrains due to numerical discretization, which are used to derive a correction displacement field that is viewed as the fine scale solution. By homogenizing the two scale, one may derive an self-adjusting weak formulation that is capable to minimize discretization error for a give mesh.
In this approach, one finds the fine scale solution by utilizing Eshelby's eigenstrain formulation or single inclusion solution. By substituting the fine scale solution into the coarse scale weak formulation, one can find a new homogenized coarse scale weak form, and subsequently, one can solve the homogenized coarse scale problem and obtains numerical solution with better accuracy.

References

1. Eshelby, J. D. [1957], "The elastic field outside an ellipsoidal inclusion," *Proceedings of Royal Society of London*, **A**252, 561-569.

2. Eshelby, J. D. [1961], "Elastic inclusions and inhomogeneities," in *Progress in Solid Mechanics*, **2**, 89-140, edited by I.N. Sneddon and R. Hill, North-Holland, Amsterdam,

3. Hughes, T.J.R. [1995], "Multiscale phenomena: Green's functions, the Dirichlet-to-Neumann formulation, subgrid scale models, bubbles and the origins of stabilized methods," *Computer Methods in Applied Mechanics and Engineering*, **127**, 387-401.

4. Hughes, T.J.R., Feijoo, G. R., Mazzei, L., and Quincy, Jean-Baptiste [1998], "The variational multiscale method – a paradigm for computational mechanics," *Computer Methods in Applied Mechanics and Engineering*, **166**, 3-24.

5. Mura, T. [1987] *Micromechanics of Defects in Solids*, Martinus Nijhoff Publishers, Dordrecht