A COMPARISON OF RFB AND NOPG FOR ACOUSTICS

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Residual-free bubbles (RFB) are a finite element approach that seeks to attain high coarse-mesh accuracy, assuring good performance of the computation at any mesh resolution [1]. This is achieved by adding a residual-free bubble enrichment to the standard polynomial field within each element. An alternative approach is to seek best approximation in the H^1 semi-norm for any operator, regardless of the related energy norm. Considering local weighting functions, for computational efficiency, leads to a nearly optimal Petrov-Galerkin formulation (NOPG) [2]. Again, this is achieved by adding a bubble enrichment to the standard polynomial field within each element.

The application of the RFB and NOPG approaches to the Helmholtz equation is identical on meshes for which the Laplacian of the underlying finite element polynomials vanishes, such as the structured meshes of linear elements that are considered in dispersion analyses. The bubble functions, which define the two methods, are solutions of element-level, homogeneous, Dirichlet boundary-value problems. In multi-dimensional configurations, these are usually represented analytically in the form of series.

We examine three series representations for the bubbles [2, 3, 4] in two dimensions. By means of dispersion analysis, we are able to determine the number of terms in each of the three series that is required to achieve a given level of accuracy. One of the series representations stands out as the most efficient.

Dispersion properties of the RFB and NOPG methods are presented. These results shed light on the high degree of accuracy for cases in which element diagonals are aligned with directions of propagation, that was observed numerically [5].

References

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