A MULTISCALE VIEW OF THE GALERKIN METHOD IN ONE DIMENSION: THE p VERSION OF THE FEM DOES NOT REQUIRE STABILIZATION

C.L. Bottasso[a] and **D. Detomi**[b]

[a]School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0150, USA
carlo.bottasso@ae.gatech.edu

[b]Dipartimento di Ingegneria Aerospaziale
Politecnico di Milano
Via La Masa 34, Milano 20158, Italy
detomi@aero.polimi.it

In this paper we study the p version of the Galerkin method, looking at it from a multiscale perspective [1]. The investigation here presented is inspired by the observation that the stabilizing mechanism provided by the sub-grid model of a multiscale approach must be contained in a p Galerkin method, if one interprets the higher order components of the finite element basis as the fast scales. This also means that, as long as the quality of the sub-grid model provided by the finite element basis is acceptable, the p Galerkin method must be a viable method that does not require any further ad-hoc modification to yield good quality solutions. This would however appear to be in contrast with the common view that the plain Galerkin method is in fact unsuitable for a variety of problems.

In order to clarify this issue, we analyze the advection-diffusion case in one spatial dimension, where the Galerkin method is known to oscillate in the advection dominated regime. The multiscale analysis shows that indeed only the higher order degrees of freedom, those modeling the fine unresolvable modes, do oscillate, while the coarse scales are free from this spurious effect. The plain p Galerkin method can therefore be safely used in these cases, as long as one recognizes the different roles played by the various degrees of freedom. This implies that the higher order modes should not be used for computing the solution, which has to be reconstructed using an appropriate post-processing (re-interpolation) of the coarse resolvable values. This is a non-standard way of interpreting a finite element solution, but it is however the key to the successful use of the Galerkin method in this class of problems. These results hold in the linear and non-linear cases.

Using the same reasoning, one can also show that the higher order modes play the additional role of built-in a-posteriori error estimators. These are computed as part of the solution process of the p method, and consequently are obtained at no additional cost.

Next, we apply the same ideas to the Helmholtz operator, and we recover well known results on the good behavior of the higher order Galerkin method in this case. Numerical examples are used throughout the paper to illustrate the main findings. Finally, we comment on the applicability of these concepts to the multi-dimensional case.

References

[1] T.J.R. Hughes, G.R. Feijóo, L. Mazzei, and J.B. Quincy, "The Variational Multiscale Method — A Paradigm for Computational Mechanics," *Computer Methods in Applied Mechanics and Engineering*, v. 166, p. 3–24, 1998.