RESIDUAL BASED COMPUTATION OF THE STABILIZATION PARAMETERS USING FINITE CALCULUS AND THE FINITE ELEMENT METHOD

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Computation of the stabilization parameters is one of the critical issues in stabilized numerical methods. Indeed the stabilization and accuracy of the numerical solution can be considerably improved by the correct evaluation of the stabilization parameters.

Finite calculus (FIC) is a general procedure to derive stabilized numerical methods. The starting point are the modified governing equations of the problem derived by accepting that the balance laws in mechanics are satisfied in a domain of finite size. The new FIC governing equations contain higher order terms than those of the infinitesimal theory. The FIC equations are the basis for deriving stabilized numerical schemes using any space and time discretization procedure [1]. Applications of the FIC method to problems in fluid and solid mechanics are reported in [1-3].

The paper shows a general iterative procedure for computing the stabilization parameters from the residuals of the FIC equations discretized using the finite element method. It is shown that for 1D situations the standard expression of the stabilization parameter for convection-diffusion problems is obtained. An iterative diminishing residual (DR) scheme is proposed for 1D problems establishing that the updated value of the stabilization parameter must ensure a reduction of the element residuals. The DR method is explained in some detail together with the procedure for computing the stabilization parameter at elements adjacent to the boundaries of the analysis domain.

The DR method is extended to 2D and 3D situations for the iterative computation of the components of the stabilization parameter vector. The split of this vector along the streamline and the solution gradient directions is essential for computing the correct stabilization parameters. It is shown that the method provides a useful scheme for advective-diffusive problems involving boundary layers and sharp internal gradients. A simple and effective approximation of the solution gradient vector in boundary elements is described.

Extensions of the DR procedure to compute the stabilization parameters in advection-diffusion-absorption problems and for incompressible problems in fluid and solid mechanics are presented together with some examples of application.

References

[1] E. Oñate, "Derivation of stabilized equations for advective-diffusive transport and fluid flow problems". *Comput. Methd. Appl. Mech. Engrg*. v. **151** (1/2), p. 233-267, 1998.

[2] E. Oñate and J. García, "A finite element method for fluid-structure interaction with surface waves using a finite calculus formulation". *Comp. Meth. Appl. Mech. Engng.*, v. 191 6-7, p. 635 –660, 2001.

[3] E. Oñate, J. Rojek, R.L. Taylor and O.C. Zienkiewicz, "Finite calculus formulation for incompressible solids using linear triangles and tetrahedra". Submitted to *Int. J. Num. Meth. Engrg.*, 2002.