# ADAPTIVE FINITE ELEMENT MODELING OF WAVE PROPAGATION IN SOLIDS

**Zhihua Yue** and **Donald H. Robbins, Jr.**

Department of Mechanical Engineering
The University of Maryland
College Park, Maryland 20742
email: drobbins@eng.umd.edu

An efficient simulation of highly transient phenomena such as wave propagation in solids requires computational techniques that are adaptive with respect to both space and time. While the use of non-adaptive, uniformly refined meshes and constant time steps simplifies model development, the computational cost of this option is prohibitive for large-scale problems of practical interest. For this class of problems, the localized and propagative character of stress waves poses a severe challenge for creating optimal spatial discretizations which must continually adapt to these rapidly moving disturbances. Most adaptive techniques have relied on the processes of adjusting the time step and repeatedly reformulating the global mesh in response to various computed temporal and spatial error estimators[1]. However, the processes of reformulating the global mesh and the subsequent re-interpolation of the solution represent a significant burden, both in terms of computational effort and model development complexity. In this paper, we simplify this process by using finite element mesh superposition as the means of achieving mesh adaptation.

Finite element mesh superposition[2] is a mesh refinement technique that creates refined subregions within an existing finite element mesh by superimposing smaller, independent, refined meshes (i.e. overlay meshes) on the original mesh to create a composite mesh. No changes are made to the original mesh during the superposition process. Furthermore, the original mesh and the overlay mesh(es) need not have compatible discretization, thus the overlay mesh(es) can be used to provide enhanced interpolation capability precisely where it is needed, regardless of the original mesh geometry. Mesh superposition is based on a hierarchical representation of the displacement field, whereby the displacement field that is interpolated on the overlay mesh serves as an incremental enhancement to the displacement field that is interpolated on the fixed original mesh. The issue of continuity of the resulting hierarchical displacement field is achieved by simply imposing homogeneous essential boundary conditions on the overlay mesh. The process of adding and removing the independently created overlay meshes is shown to be both efficient and accurate, and its simplicity permits the process to be fully automated with minimal code development complexity.

While mesh superposition has been used successfully in static analyses, its utility in transient analysis has not been previously explored. The current research effort includes the initial extension of the technique to transient problems and investigates several issues pertinent to its efficient application in wave propagation analysis. Results obtained with uniformly refined meshes and constant time steps are compared to results obtained using several variants of the mesh superposition concept, in conjunction with different spatial error estimators. Moreover, several iterative solution methods, such as the PCG Method and Multi-Grid Method are utilized in an attempt to efficiently solve the special block-structured composite systems arising from the mesh superposition technique. The resulting methodology permits adaptation of the composite mesh and time step to be fully automated so that both spatial and temporal discretization errors are controlled within specified tolerances.

**References**

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