PHASE FIELD METHOD IN OPTIMAL DESIGN

B. Bourdin[a] and **A. Chambolle**[b]

[a]Department of Mathematics, Louisiana State University,
Baton Rouge, LA 70803, USA. bourdin@math.lsu.edu

[b]CEREMADE, Université Paris Dauphine, Place du Marchal De Lattre De Tassigny,
75775 Paris CEDEX 16, France. chambolle.ceremade.dauphine.fr

The phase field method is a versatile, robust, and rigorous framework for topology optimization problems. It is based on the penalization of the variation of the properties of designs (*i.e.* perimeter penalization), and its variational approximation. It uses a smooth function, the phase-field, to represent the materials involved in the device or the system.

Consider the following optimal design problem of finding p materials occupying p disjoint regions $D_1, \dots, D_p$ of a ground domain Ω, and minimizing the objective function F, under a perimeter constraint:

$$(\mathcal{P}): \inf_{D_1,\dots,D_p \text{ admissible}} F(D_1,\dots,D_p) + \sum_{1 \le i \le j \le p} \mathit{length}(\partial D_i \cap \partial D_j).$$

In the phase field framework, one uses a single differentiable function $\rho = (\rho_1, \dots, \rho_p)$ to represent all the materials. For any $\varepsilon > 0$, one defines the following problem:

$$(\mathcal{P}_\varepsilon): \inf_\rho F_\varepsilon(\rho) + \int_\Omega \frac{1}{\varepsilon} W(\rho) + \varepsilon |\mathrm{D}\rho| \, dx,$$

where W is a p–wells function, such that $W(\rho) = 0$ if one and only one of the components of ρ is equal to 1, and strictly positive otherwise. Then, under some technical conditions on F, one can prove that when $\varepsilon \to 0$, the solutions of problem $(\mathcal{P}_\varepsilon)$ converge in some sense to that of $(\mathcal{P})$. Moreover, since $(\mathcal{P}_\varepsilon)$ is a well-posed problem, whose arguments are classical differential functions, the convergence result suggests a numerical algorithm, that is to solve $(\mathcal{P}_\varepsilon)$ for a "small enough" ε.

This framework has already been applied to several problems in structural optimization, including the stiffness optimization of pressurized structures, as illustrated in the following figure.

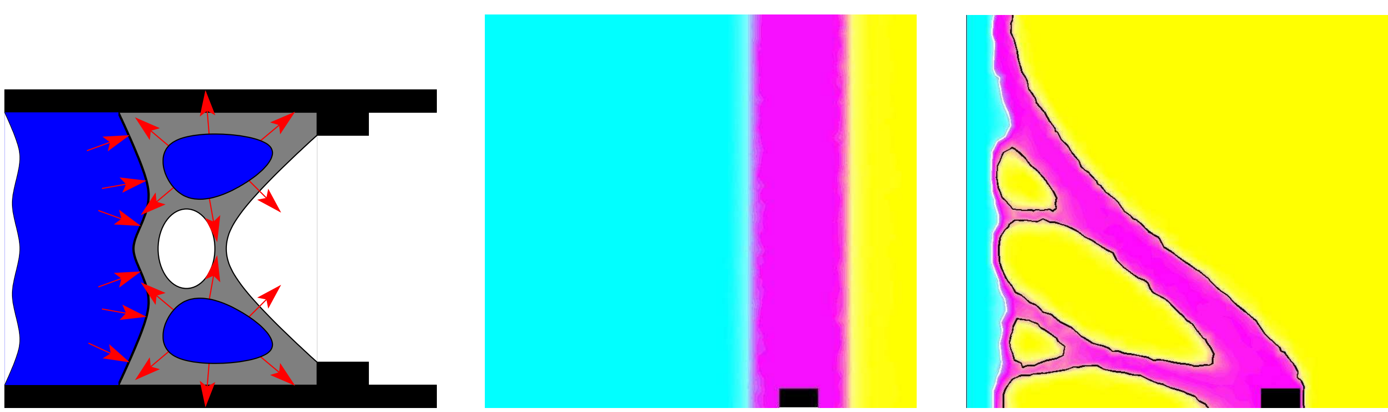

Figure 1: Optimal design of pressurized structure. From left to right: schematic of the problem, phase field ρ for the initial design on a half domain (blue corresponds to a liquid under pressure, magenta to some elastic material and yellow to the void), and the final design.

In this talk, we will briefly present the general framework, detail its application to the stiffness optimization