CONVERSION OF IMPERFECTION-SENSITIVE TO -INSENSITIVE ELASTIC STRUCTURES

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Many engineering structures are imperfection-sensitive. Imperfection sensitivity is defined by specific mathematical characteristics of the post-buckling path at the stability limit. In case of symmetric bifurcation, the mechanical consequence of these characteristics is loss of stability of an imperfect structure by snap-through at a load level that may be significantly lower than the one at the bifurcation point of the perfect structure. From the viewpoint of engineering design, a conversion from an originally imperfection-sensitive to an imperfection-insensitive structure is desirable. How to modify the original design of the structure to achieve such a conversion? That is the question which will be addressed in this paper.

Koiter's initial post-buckling analysis is employed in the context of the Finite Element Method to derive a relationship between three parameters – λ_2, λ_4, a_1 – playing a central role in case of symmetric bifurcation [1], [2]. The parameter λ_2 is related to the initial slope, whereas the parameters λ_2 and λ_4 are related to the initial curvature of the post-buckling path at the bifurcation point. The third parameter, a_1, is closely related to the curvature of the eigenvalue curve, at the stability limit, based on the consistently linearized eigenvalue problem [3], [4]. It is shown that the case of interest, i.e., $\lambda_2 = 0$, is characterized either $a_1 = 0$ or $1/a_1 = 0$ and by special mathematical properties of the eigenvalue curve and the corresponding eigenvector curve, both depending on the load parameter. The common feature of the eigenvector curves at $\lambda_2 = 0$ is the existence of a singular point of the vector function $\mathbf{u}_1^*(\lambda)$ at the stability limit, reflecting a fundamental change of the mechanical behavior of the structure. Ten different modes of transition are shown to exist. Only three of them include a change of the sign of λ_2, leading to a conversion from imperfection sensitivity to insensitivity.

Two relatively simple numerical examples serve as the vehicle to corroborate the theoretical findings. Different forms of stiffening of structures such as, e.g., appropriate attachment of a spring or the change of the thickness or height of the structure, are considered.

References

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