

# Multiscale Turbulence Modeling in the Discontinuous Galerkin Method

S. Scott Collis<sup>a</sup>, Kaveh Ghayour<sup>b</sup>, and Srinivas Ramakrishnan<sup>c</sup>

<sup>a</sup>Department of Mechanical Engineering and Materials Science  
Rice University  
Houston, TX 77005-1892  
collis@rice.edu

<sup>b</sup>Department of Mechanical Engineering and Materials Science  
Rice University  
Houston, TX 77005-1892  
kaveh@rice.edu

<sup>c</sup>Department of Mechanical Engineering and Materials Science  
Rice University  
Houston, TX 77005-1892  
seenu@rice.edu

While notable progress has been made in recent years on extending Large Eddy Simulation (LES) to complex geometries for incompressible flows, developments for compressible flows have lagged behind. In this talk we present a method that exploits the flexibility and accuracy of a discontinuous Galerkin method (DGM) with the Variational Multi-Scale (VMS) approach to LES. The combination of DGM and VMS holds particular promise for LES of compressible turbulent flows in complex geometries.

It is generally recognized that high-order spatial discretizations are advantageous when performing Direct Numerical Simulation (DNS) or LES of turbulent flows. In order to obtain higher-order accuracy, many numerical methods traditionally used for DNS and LES, such as finite-difference and spectral methods, directly enforce higher regularity in the computed solution through the use of wide stencils or smooth basis functions. Unfortunately, this requirement for higher regularity often comes at the expense of flexibility. Here, we take a different approach by applying the discontinuous Galerkin method (DGM) to LES of compressible, turbulent flows. In DGM, the computed solution is discontinuous between element interfaces with adjacent elements communicating through appropriate numerical fluxes similar to those used in finite-volume methods. High order accuracy is achieved by locally enriching the function space on each element.

Because it is based on a variational formulation with relatively rich hierarchical solution representations on each element, the DGM is particularly well suited for the VMS approach to LES introduced by Hughes et al. [1]. The combination of DGM and VMS provides extensive flexibility that can be exploited for LES in a number of ways including: unstructured meshes, local  $hp$ -refinement, and hybrid models. These potential features make DGM with VMS particularly attractive for LES of compressible turbulent flows in complex engineering geometries.

This talk focuses on the multiscale formulation of DGM as well as preliminary results for canonical compressible flows including channel flow and the wake of a circular cylinder in crossflow.

## References

[1] T.J.R. Hughes, L. Mazzei, and K.E. Jansen, “Large Eddy Simulation and the Variational Multiscale Method,” *Computing and Visualization in Science*, v. 3, p. 47–59, 2000.