UNIFORM SAMPLE DISTRIBUTIONS FOR OPTIMIZATION AND UNCERTAINTY

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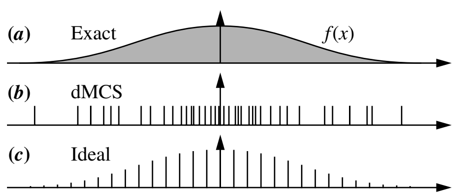

Figure 1. Ideal realization weights (adapted from [1]).

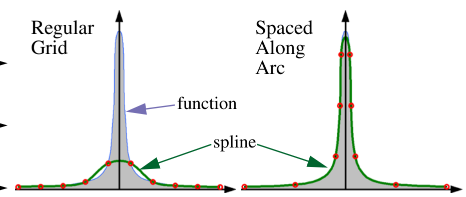

Figure 2. Ten evenly-spaced sample points to approximate an input/output map.

Approximately uniform distributions of sample points are useful in certain problems in engineering and mathematics, notably optimization and uncertainty. For example, weight-controlled Monte Carlo simulation (MCS), a technique to improve estimates of low-probability events, would ideally have samples approximately uniformly distributed throughout the sample space with weights estimating probability. Response surface methodologies likewise benefit from an even distribution of sample points, not only over the input space but also over the output space, particularly in regions where steep gradients occur. The notion of uniformity can be quantified by the concept of *discrepancy*, which is a measure of the difference between a sample distribution and an ideal distribution — in this case, a uniform distribution. This paper advances the *discrepancy sensitivity*, a linear approximation for the change in discrepancy when adding or removing a particular sample point, as a numerical measure of a sample point's contribution to uniformity. Essentially, this quantifies the level of "new" information provided by a sample point. Several examples are given of its use in choosing sample points for response surface characterization. The example problems, while of trivial computational complexity, exhibit some of the "difficult" behaviors — such as multimodality, sharp corners, and so forth — that are typical of many real world applications. It is demonstrated that using the discrepancy sensitivity, sample points can be effectively spread throughout the "interesting" regions of these functions. Some indications of the current work-in-progress and future directions are given.

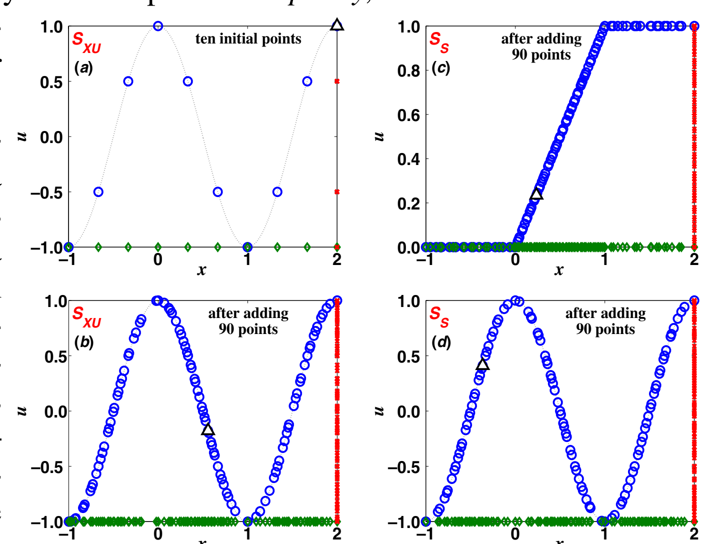

Figure 3. Sample point distribution (blue circles) and the distributions projected just in the input space (green diamonds) and just in the output space (red x's) for two different functions and two methods.

References

[1] Pradlwarter, H.J., and G.I. Schuëller, "On Advanced Monte Carlo Simulation Procedures in Stochastic Structural Dynamics," *International Journal of Non-Linear Mechanics*, v. 32, pp. 735–744, 1997.

* Sandia is a multiprogram laboratory operated by the Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000.