Galerkin Finite Element Methods for Stochastic Elliptic Partial Differential Equations

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We consider numerical approximations for the statistical moments of the solution of a model linear elliptic partial differential equation with stochastic coefficients, illustrating on the particular case of the expected value and estimating numerical and modeling errors. In particular, we present a priori error estimates for different numerical methods.

The stochastic coefficients are approximated by truncated Karhunen-Loève expansions driven by a finite number of uncorrelated random variables. This approximation can be seen as a perturbation of the initial stochastic elliptic problem and a modeling error is then introduced. For this reason we estimate the perturbation on the solution of the initial stochastic elliptic problem in terms of perturbations on the stochastic coefficients. After approximating the stochastic coefficients the original stochastic elliptic problem turns into a new deterministic parametric elliptic problem. The dimension of the parameter equals the number of random variables needed to approximate the stochastic coefficients in the original problem.

We describe and analyze two numerical approximations for the expected value of the solution. The first is the Monte Carlo Galerkin Finite Element Method (MCGFEM) which generates iid standard Galerkin finite element approximations of the solution by sampling the stochastic coefficients of the equation. The second, the so called k-h version of the Stochastic Galerkin Finite Element Method (k-h SGFEM), uses a Galerkin variational formulation based on a tensor product of the space consisting of continuous piecewise linear functions over an h-triangulation of the domain of the elliptic problem, and of the space consisting of piecewise polynomials over a k-partition of the parameter set. For both methods, we present a priori error estimates in the H^1 and L^2 norm for the convergence of the numerical approximations towards the expected value of the solution.

References

[1] I. Babuška , R. Tempone and G.E. Zouraris, “Galerkin finite element approximations of stochastic elliptic partial differential equations,” *TICAM Report 02-38,* 2002.