The p-h version of the Stochastic Galerkin FEM: Error Analysis and Computational Efficiency Comparisons

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This talk considers numerical approximations of the statistical moments of the solution of a model linear elliptic equation with stochastic coefficients, illustrating on the particular case of the expected value and estimating discretization errors. In particular, we are interested in the computational work required by different numerical approximations to achieve a prescribed accuracy.

The stochastic coefficients are approximated by truncated Karhunen-Loève expansions driven by a finite number of uncorrelated random variables. Following this line, the original stochastic elliptic problem turns into a deterministic parametric elliptic problem. The dimension of the parameter equals the number of random variables needed to approximate the stochastic coefficients in the original problem.

Since the solution of the parametric elliptic problem is analytic with respect to the parameter variable we propose a tensor product approximation, the p-h version of the Stochastic Galerkin Finite Element Method (SGFEM), that uses a p version on the parameter set and an h version on the physical domain. We prove a priori error estimates in the H^1 and the L^2 norm, yielding exponential convergence with respect to the order p.

To compute efficiently the solution of the p-h SGFEM we use a special basis with double orthogonal polynomials. Thus, finding the solution entails solving a number of standard elliptic problems corresponding to different values of the parameter, precisely like in the Monte Carlo method.

Finally, using the available a priori error estimates for the methods Monte Carlo Galerkin Finite Element Method (MCGFEM), k-h SGFEM and p-h SGFEM, we compare the asymptotic computational work required by each method to achieve a given accuracy. This comparison suggests intuitive conditions for an optimal selection of these methods.

References

[1] I. Babuška, R. Tempone and G.E. Zouraris, "Galerkin finite element approximations of stochastic elliptic partial differential equations," *TICAM Report 02-38,* 2002.