VARITIONALLY CONSISTENT HIGHER ORDER GLOBAL DIFFERENTIABILITY FINITE ELEMENT PROCESSES FOR ELASTIC WAVE PROPAGATION IN LAMINATED COMPOSITES

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This paper utilizes a new mathematical and computational framework [1] for treating initial value problems (IVP) of elastic wave propagation in mediums with bi-material interfaces. Two essential elements of the proposed mathematical and computational framework [1] are (1) the space-time *variational consistency* (STVC) of the mathematical approach used in obtaining algebraic systems from the differential systems via a space-time integral approach, and (2) the *global differentiability* (or smoothness) of the space-time local approximations requiring the use of higher-order spaces in space and time (*k*-version of finite element method). The STVC is a fundamentally important aspect of the proposed approach. A violation of this leads to space-time variationally inconsistent (STVIC) process that may be degenerate. The degree of global differentiability in space and time is of critical importance: (i) If the approximations are to be admissible in the weak form; (ii) If the approximations are to be admissible in the governing differential equations (GDE) (iii) if the approximations are to yield the same global smoothness as the analytical solutions; and (iv) to permit correct incorporation of the physics of the processes in the numerical computations.

One dimensional elastic wave propagation in a laminated composite medium containing bi-material interfaces is used as a model problem to demonstrate that (1) space-time Galerkin method with or without weak forms are STVIC and hence would lead to degenerate computational processes. (2) Quasi-methods in which space and time are decoupled, do not permit construction of STVC integral forms. They suffer from stability and accuracy issues and hence do not permit true time evolutions. (3) Space-time Least Squares process (STLSP) utilizing strong form of the governing differential equations yield STVC space-time integral forms and hence are meritorious in all aspects over all other methods. (4) STLSP permit the solution to be marched in time i.e. one only needs to consider one time increment at a time. (5) Bi-material interfaces and accountability of more realistic physical material behavior at the interface are of critical significance in computing evolutions that are physical. (6) The solutions of the problems under consideration are undoubtedly of higher order global differentiability. That is, even though for the model problem the GDE is a second order partial differential equation in space and time but the theoretical solutions may be of higher classes in space and time. Such solutions necessitate the use of higher order Hilbert spaces $\hat{H}^{k,p}(\overline{\Omega}_{xt})$, $k=(k_1,k_2)$; k_1 and k_2 being the orders of space in space and time for the local approximations. (7) It is shown that the process of wave reflection and transmission at the interface is highly dependent on the material properties at the interface and in its immediate vicinity. An approach to account for interface material properties is presented in which higher order global differentiability of the local approximations yields higher order global differentiability of the computed solutions. Numerical studies are presented for 1D elastic wave propagation in laminated composites. Superiority of approach proposed in this paper is clearly demonstrated.

References

[1] K.S. Surana, J.N. Reddy, S. Allu, "Variationally Consistent *k*-version Finite Element Processes for Initial Value Problems," (*in preparation*)