A VELOCITY POST-PROCESSING FOR THE HELMHOLTZ EQUATION

Rivania H.Paulino de Romero and **Fernando Alves Rochinha**

Department of Mechanical Engineering
Federal University of Rio the Janeiro
Rio de Janeiro, Brasil, 21945-970
faro@serv.com.ufrj.br

In acoustic problems, the steady state sound response in a non viscous medium is governed by the Helmohtlz equation [2] . One major problem in numerically solving this equation by means of the Finite Element Method(FEM) is linked to a potential loss of ellipticity with the increasing of the excitation frequency, which directly impairs the convergence of the acoustic velocity.

A Macroelement Post-Processing recovery technique [1] of the acoustic velocity field based on residuals of equilibrium equation and irrotational condition for the Helmholtz equation is proposed. The derivatives are recovered by solving local variational problems exploring special superconvergence points. The basic idea of the method is to find a better approximation for the derivative (∇p_h) by solving a local variational problem on each macroelement, understood as the union of a set of neighboring elements with common edges.

To this end, one can suppose that the domain Ω is decomposed into macroelements not necessarily disjoints and define $MP_h^r \subset \mathcal{L}^2(\Omega)$ and $MQ_h^r \subset \mathcal{L}^2(\Omega)$, respectively, triangular and quadrilateral lagrangian finite element spaces of piecewise polynomial of degree r on each element and class C^0 in each macroelement but discontinuous on the macroelement boundaries. Considering $Z_h = MP_h^r \times MP_h^r \times MP_h^r$ or $Z_h = MQ_h^r \times MQ_h^r \times MQ_h^r$, the following residual form for the problem of retrieving a better approximation of the 3D acoustic velocity field is proposed PROBLEM ME_h: Given p_h, a finite element approximation of the pressure field, find the post-processed velocity field $\mathbf{v}_h^* \in Z_h$, such that

$$b_h(\mathbf{v}_h^*, \mathbf{u_h}) = (\frac{i}{w\rho}\nabla p_h, \mathbf{u_h})_h - \delta_1 h^2(\frac{iw}{\rho c^2} p_h, div\mathbf{u_h}) \quad \forall\ \mathbf{u_h} \in Z_h \tag{1}$$

where the sesquilinear form b_h is defined as

$$b_h(\mathbf{v}_h^*, \mathbf{u}_h) = (\mathbf{v}_h^*, \mathbf{u}_h)_h + \delta_1 h^2(div\mathbf{v}_h^*, div\mathbf{u}_h) + \delta_2 h^2(curl\mathbf{v}_h^*, curl\mathbf{u_h}) \tag{2}$$

where $\mathbf{v}_h^*$ is the post-processed velocity. The scalars δ_1 and δ_2 are positive real parameters, that might be adjusted in order to have better convergence performance. A number of comprehensive simulation is presented in order to access the main features of the proposed methodology.

References

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[2] Ph. Bouilarrd and F. Ihlenburg, "Error estimation and adaptivity for the finite element method in acoustics: 2D and 3D applications,"*Computational Methods Applied Mechanical Engineering*, v. 176, p. 147-163, 1999.