HIGH ORDER NODAL DG-FEM FOR THE MAXWELL EIGENVALUE PROBLEM

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Accurate, efficient, and robust methods for solving the eigenvalue problem associated with the time-harmonic vectorial Maxwell equations continue to be of interest in many areas of computational electromagnetics (CEM). Unfortunately, the discretization of the curl-curl operator holds more challenges than meets the eye. Indeed, as has been known for 3 decades, the straightforward use of C^0 nodal FEM fails in several ways. On general unstructured grids such formulations are unable to correctly represent the null-space of the operator. Furthermore, the dispersion relation of the C^0 FEM discretization results in seemingly converging spurious eigensolutions, often indistinguishable from physical solutions. The remedy to all these troubles are by now classical and involves the use of curl-conforming elements.

In this work we shall explore the use of discontinuous Galerkin finite element methods (DG-FEM) for the solution of the Maxwell eigenvalue problem. Fully unstructured grids using high-order Lagrangian elements are being used. The construction of the scheme is discussed and it is pointed out that by choosing the numerical fluxes correctly, the formulation is weakly curl-conforming. Furthermore, we show that the numerical dispersion relation is such for the DG-FEM that seemingly convergent single eigenvalues can be ruled out. This leaves the proper representation of the null-space of the curl-curl operator as the only remaining concern. For the two-dimensional case, we resolve this by establishing an exact connection between the discrete curl-curl form and the Helmholtz problem. Relying on consistency and coercivity of the latter suffices to guarantee convergence of the eigenvalues. Several examples confirm these results and show the close connection between the regularity of the eigensolution and the h/p-convergence rate.

For the three-dimensional case, a naive use of the local DG-FEM discretization fails to correctly represent the null space of curl-curl operators and spurious low-frequency solutions reappear. We observe that the connection between the discrete curl-curl operator and the Helmholtz equation is no longer exact – a key difference from the two-dimensional case. We shall discuss, however, how a minor modification of the DG-FEM fluxes suffices to overcome the problem. The success of this is illustrated through a few examples.

The use of the DG-FEM with nodal elements has a number of advantages over the more traditional formulation based on curl-conforming elements. Most notably is that the latter results in a generalized eigenvalue problem while the former results in a simpler standard symmetric eigenvalue problem. Furthermore, the DG-FEM method has proven itself very powerful for solving Maxwell's equations in the time-domain. The developments reported here suggest that a unified time-domain/frequency-domain formulation is possible and likely to yield an accurate, efficient, and robust scheme for a variety of problems in CEM.