A FAST MULTI-LEVEL TIME CONVOLUTION METHOD FOR TRANSIENT HEAT DIFFUSION

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Over the past three decades, boundary element methods (BEM) have been widely used to numerically solve time-dependent problems in engineering and science. For a variety of problems, the use of the time-convolution approach [1] facilitates a boundary-only discretization of the domain. However, the algorithm requires a re-evaluation of the time convolution integrals at each time step. In general, the time-convolution approach leads to the computational complexity of order $O(M^2N^2)$, where M is the number of degrees of freedom due to boundary element discretization, and N is the number of time steps. Furthermore, the memory requirements for the conventional time-convolution approach are also of order $O(M^2N^2)$. When N is relatively small and the problem is two-dimensional, the conventional time-convolution BEM are fast and, thus, acceptable. However, for large numbers of time steps, e.g. $N > 1000$, the conventional time-convolution approaches become prohibitively expensive even on the state-of-the-art supercomputers. For the three-dimensional BEM formulations involving very large numbers of degrees of freedom M, the conventional time-convolution algorithm becomes impractical when more than 100 time steps are required.

In this presentation, we develop a novel fast multi-level time-convolution algorithm based on the pioneering work of Brandt and Lubrecht [2] on the multi-level multi-integration (MLMI). Most recently, Grigoriev and Dargush [3] proposed a fast multi-level boundary element method (MLBEM) for the Laplace equation that extends the MLMI algorithm [2] for the solution of mixed boundary value problem of steady heat diffusion equation. It has been shown [3] that the CPU and memory requirements for the fast MLBEM algorithm can be orders of magnitude less than for the conventional BEM. Here, we propose a fast multi-level algorithm for the time-convolution and demonstrate that both the CPU and memory requirements can be as low as $O(M^2N)$.

The attractiveness of the fast multi-level time-convolution algorithm is demonstrated for a transient heat diffusion problem in two-dimensions that possesses an exact solution. Figure 1 shows the time history of the error $\|T\|$ for the conventional and fast time-convolution boundary element algorithms. Notice that the error for the fast convolution can be as low as that for the conventional algorithm provided that a sufficient number of points are retained in the singularity circle R_s. Comparisons of the CPU time and memory requirements for the conventional and fast algorithms are provided in Table 1.Notice that both the CPU time and memory requirements for the fast convolution algorithm is very favorable even for the modest number of time steps, $N = 1000$.

Algorithm	Memory, Mb	CPU time, min
Conventional	518	555
Fast, $R_s = 17$	20	30

Table 1: CPU time and memory requirements.

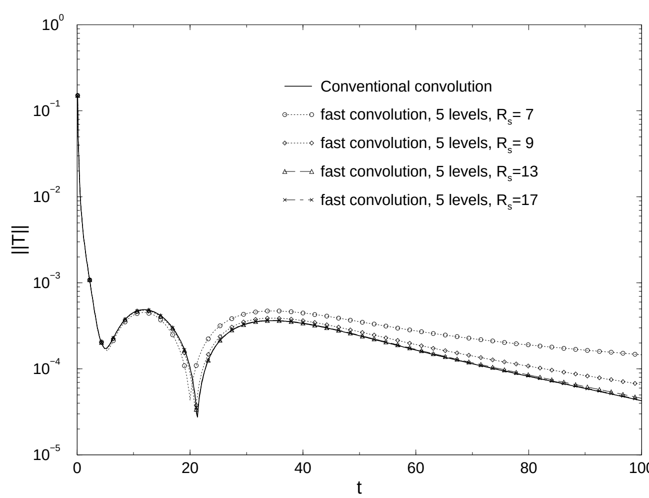

Figure 1. Time history of the solution error.

References

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