MESHES FOR CHARACTERISTIC LENGTH SCALES

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Solutions of singularly perturbed problems can often be viewed as linear combinations of solutions with characteristic length scales[1]. Usually the solution consists of a smooth component with length scale equal to the diameter of the computational domain and boundary layers each with a parameter-dependent length scale. For certain classes of singularly perturbed problems, the layer structure is very rich and the layers are not necessarily confined to the boundaries of the domain. This is especially the case with thin shell problems[2] where sources of layers include for instance load irregularities. The philosophy we adopt is to use higher-order finite elements to resolve the smooth component and the *hp*-approach for the layers.

For simple problems feature-based mesh generation is sufficient. However, the scales may vary as the boundary conditions and loading change. Automatic mesh generation is needed to handle this complexity. Unfortunately the standard mesh generation algorithms are not well-suited for layer adaptation which is one-dimensional. In ideal case the mesh lines are perfectly aligned to every layer inducing feature of the problem and with appropriate grading. Our approach is a hybrid method where we attempt to utilize a priori knowledge of the solution while making it possible to discover a posteriori features of the solution.

The key idea is that we let every source of layers to induce the necessary mesh lines with one-dimensional grading. Thus, when the actual mesh generation starts, the input is a collection of the boundary information and the induced "virtual boundaries." The feature or patch structure is formed from this information by computing the so-called arrangement of the line segments[3]. Every cell in an arrangement corresponds to a patch of the mesh which can then further be divided into smaller elements if necessary. It should be noted that in simple cases, rectangular domain with a single boundary layer, say, the arrangement is the minimal optimal mesh.

Of course one must identify the sources of layers for this scheme to work, so here adaptivity means feature discovery. The purpose of a posteriori error analysis is to guide the creation of the virtual boundaries rather than indicate where the mesh should be refined. Refinement strategies are not successful unless the alignment requirements are properly taken into account. Therefore, we prefer to remesh at every step. For thin shell problems adaptivity is very much work in progress, however, and the effectiveness of our solver is dependent on the quality of the a priori scale resolution.

Our current implementation is based on CGAL[4] with support for arrangements of segments and circular arcs.

References

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