A TENSORIAL MODAL EXPANSION FOR TRIANGLES AND TETRAHEDRA

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The p and hp version of the FEM have been extensively used in the solution of engineering problems. The effective use of these techniques relies on many factors and the selection of appropriate hierarchical or modal basis functions is very important. In general, the functions are associated to topological entities of the finite elements, i.e., vertex, edge, face and body.

The most used hierarchical basis functions for triangles and tetrahedra do not rely on tensor product of one-dimensional polynomials [1,2,3]. One noticeable exception is the modal functions based on Jacoby polynomials developed in [4] and used in the Spectral/hp FEM. This basis uses a triangle to quadrangular mapping to define the interpolation functions trough the tensor product of unidimensional Jacobi polynomials.

In this work, modal basis functions for triangles and tetrahedra based on the tensor product of Jacoby polynomials expressed in terms of baricentric coordinates are presented. The procedure to construct the expansion is introduced using Lagrange polynomials to obtain the standard h functions for triangles. After that, the modal basis functions are determined. One of the main aspects of the modal basis expansion is that global continuity of the approximation is achieved by using the standard superposition procedure of the element matrices. It is not necessary to multiply the odd modes by -1. This features allows the uniform treatment of h, p and hp adaptive refinements.

Results are presented to illustrate the approximation, sparsity and conditioning properties of the modal basis functions obtained.

References

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