MULTILEVEL HP FINITE ELEMENT METHODS

G. Zumbusch[a]

[a]Institut für Angewandte Mathematik
Friedrich-Schiller-Universität Jena
07740 Jena, Germany
zumbusch@math.uni-jena.de

An important tool in engineering is the finite element method. In order to increase, to reduce the computational effort, and to control the error of the numerical solution, adaptive grid refinement has been introduced. Both smaller elements (h-version) and higher order approximations (p-method, pseudo-spectral method) are common. The combination of both methods, the *h-p* version, supplies the pre-asymptotic exponentially convergent *p*-version continuously with properly adapted grids. Hence it achieves the superior exponential convergence asymptotically, too, instead of algebraic convergence of its ingredients the *h*-version and the *p*-version. Although the first theoretical results claiming these convergence rates are quite classic, the number of codes using the *h-p*-version of finite elements is still rather limited. Reasons for that are the pure implementational complexity and the details, in conjunction with the rumor of engineers' low precision requirements. But the major reason is the lack of a robust (self-) adaptive control delivering the desired exponential convergence.

Some steps towards an efficient implementation of the theoretically known exponential convergence are presented. As it turns out, an efficient implementation requires additional theoretical considerations, which play a major role there as well. This includes both the fully automatic *h-p*-version and as a subset the *p*-version on suitable grids.

References

[1] G. W. Zumbusch. *"Simultanous h-p Adaptation in Multilevel Finite Elements,"* PhD thesis, FU Berlin, 1995